

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

- 1           1. (Currently amended) A method for using a computer system to solve a  
2   system of nonlinear equations specified by a vector function,  $\mathbf{f}$ , wherein  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$   
3   represents  $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$ , wherein  $\mathbf{x}$  is a vector  $(x_1, x_2,$   
4    $x_3, \dots x_n)$ , the method comprising:  
5           receiving a representation of an interval vector  $\mathbf{X} = (X_1, X_2, \dots, X_n)$ ,  
6   wherein for each dimension,  $i$ , the representation of  $X_i$  includes a first floating-  
7   point number,  $a_i$ , representing the left endpoint of  $X_i$ , and a second floating-point  
8   number,  $b_i$ , representing the right endpoint of  $X_i$ ;  
9           for each nonlinear equation  $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$  in the system of  
10   equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ , symbolically manipulating  $f_i(\mathbf{x}) = 0$  within the computer system  
11   to solve for any invertible term,  $g(x'_j)$ , thereby producing a modified equation  
12    $g(x'_j) = h(\mathbf{x})$ , wherein  $g(x'_j)$  ~~can be~~ is analytically inverted to produce an inverse  
13   function  $g^{-1}(\mathbf{y})$ ;  
14           substituting the interval vector  $\mathbf{X}$  into the modified equation to produce the  
15   equation  $g(X'_j) = h(\mathbf{X})$ ;  
16           solving for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and  
17           intersecting  $X'_j$  with the vector element  $X_j$  to produce a new interval vector  
18    $\mathbf{X}^+$ ;  
19           wherein the new interval vector  $\mathbf{X}^+$  contains all solutions of the system of  
20   equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  within the interval vector  $\mathbf{X}$ , and wherein the width of the new  
21   interval vector  $\mathbf{X}^+$  is less than or equal to the width of the interval vector  $\mathbf{X}$ .

1           2. (Original) The method of claim 1, further comprised of performing an  
2 interval Newton step on  $\mathbf{X}$  to produce a resulting interval vector,  $\mathbf{Y}$ , wherein the  
3 point of expansion of the interval Newton step is a point,  $\mathbf{x}$ , within the interval  
4 vector  $\mathbf{X}$ , and wherein performing the interval Newton step involves evaluating  
5  $\mathbf{f}(\mathbf{x})$  using interval arithmetic to produce an interval result  $\mathbf{f}^I(\mathbf{x})$ .

1           3. (Original) The method of claim 2, further comprising:  
2           evaluating a first termination condition, wherein the first termination  
3 condition is TRUE if,  
4                               zero is contained within  $\mathbf{f}^I(\mathbf{x})$ ,  
5                                $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular, wherein  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is the Jacobian of the  
6                               function  $\mathbf{f}$  evaluated as a function of  $\mathbf{x}$  over the interval vector  $\mathbf{X}$ ,  
7                               and  
8                                $\mathbf{Y}$  contained within  $\mathbf{X}$ ; and  
9           if the first termination condition is TRUE, terminating and recording  
10  $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$  as a final bound.

1           4. (Original) The method of claim 3, further comprising determining if  
2  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular by computing a pre-conditioned Jacobian,  $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$ ,  
3 wherein  $\mathbf{B}$  is an approximate inverse of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ , and then solving for  
4 the interval vector  $\mathbf{Y}$  that contains the value of  $\mathbf{y}$  that satisfies  $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$ ,  
5 where  $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$ .

1           5. (Original) The method of claim 4, further comprising applying term  
2 consistency to  $\mathbf{B}\mathbf{f}(\mathbf{x}) = 0$ .

1           6. (Original) The method of claim 1, wherein if no termination condition is  
2   satisfied, the method further comprises returning to perform an interval Newton  
3   step on the interval vector  $\mathbf{Y}$ .

1           7. (Original) The method of claim 6, wherein returning to perform the  
2   interval Newton step on the interval vector  $\mathbf{Y}$  can involve splitting the interval  
3   vector  $\mathbf{X} = \mathbf{Y} \cap \mathbf{X}$ .

1           8. (Original) The method of claim 2, further comprising:  
2   evaluating a second termination condition;  
3   wherein the second termination condition is TRUE if a function of the  
4   width of the interval vector  $\mathbf{X}$  is less than a pre-specified value,  $\epsilon_X$ , and the  
5   absolute value of the function,  $\mathbf{f}$ , over the interval vector  $\mathbf{X}$  is less than a pre-  
6   specified value,  $\epsilon_F$ ; and  
7   if the second termination condition is TRUE, terminating and recording  $\mathbf{X}$   
8   as a final bound.

1           9. (Original) The method of claim 1, wherein for each term,  $g(x_j)$ , that can  
2   be analytically inverted within the equation  $f_i(\mathbf{x}) = 0$ , the method further  
3   comprises:  
4   setting  $X_j = X_j^+$  in  $\mathbf{X}$ ; and  
5   repeating the process of symbolically manipulating, substituting, solving  
6   and intersecting to produce the new interval vector  $X_j^+$ .

1           10. (Original) The method of claim 1, wherein symbolically manipulating  
2    $f_i(\mathbf{x}) = 0$  involves selecting the invertible term  $g(x_j)$  as the dominating term of the  
3   function  $f_i(\mathbf{x}) = 0$  within the interval vector  $\mathbf{X}$ .

1           11. (Currently amended) A computer-readable storage medium storing  
 2   instructions that when executed by a computer cause the computer to perform a  
 3   method for using a computer system to solve a system of nonlinear equations  
 4   specified by a vector function,  $\mathbf{f}$ , wherein  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  represents  $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0,$   
 5    $f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$ , wherein  $\mathbf{x}$  is a vector  $(x_1, x_2, x_3, \dots x_n)$ , the method  
 6   comprising:  
 7           receiving a representation of an interval vector  $\mathbf{X} = (X_1, X_2, \dots, X_n)$ ,  
 8   wherein for each dimension,  $i$ , the representation of  $X_i$  includes a first floating-  
 9   point number,  $a_i$ , representing the left endpoint of  $X_i$ , and a second floating-point  
 10   number,  $b_i$ , representing the right endpoint of  $X_i$ ;  
 11           for each nonlinear equation  $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$  in the system of  
 12   equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ , symbolically manipulating  $f_i(\mathbf{x})=0$  within the computer system  
 13   to solve for any invertible term,  $g(x'_j)$ , thereby producing a modified equation  
 14   |  $g(x'_j) = h(\mathbf{x})$ , wherein  $g(x'_j)$  ~~can be~~ is analytically inverted to produce an inverse  
 15   function  $g^{-1}(\mathbf{y})$ ;  
 16           substituting the interval vector  $\mathbf{X}$  into the modified equation to produce the  
 17   equation  $g(X'_j) = h(\mathbf{X})$ ;  
 18           solving for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and  
 19           intersecting  $X'_j$  with the vector element  $X_j$  to produce a new interval vector  
 20    $\mathbf{X}^+$ ;  
 21           wherein the new interval vector  $\mathbf{X}^+$  contains all solutions of the system of  
 22   equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  within the interval vector  $\mathbf{X}$ , and wherein the width of the new  
 23   interval vector  $\mathbf{X}^+$  is less than or equal to the width of the interval vector  $\mathbf{X}$ .

1           12. (Original) The computer-readable storage medium of claim 11,  
 2   wherein the method further comprises performing an interval Newton step on  $\mathbf{X}$  to  
 3   produce a resulting interval vector,  $\mathbf{Y}$ , wherein the point of expansion of the  
 4   interval Newton step is a point,  $\mathbf{x}$ , within the interval vector  $\mathbf{X}$ , and wherein

5 performing the interval Newton step involves evaluating  $f(\mathbf{x})$  using interval  
6 arithmetic to produce an interval result  $f^I(\mathbf{x})$ .

1           13. (Original) The computer-readable storage medium of claim 12,  
2 wherein the method further comprises:  
3           evaluating a first termination condition, wherein the first termination  
4 condition is TRUE if,  
5                           zero is contained within  $f^I(\mathbf{x})$ ,  
6                            $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular, wherein  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is the Jacobian of the  
7 function  $\mathbf{f}$  evaluated as a function of  $\mathbf{x}$  over the interval vector  $\mathbf{X}$ ,  
8 and  
9                            $\mathbf{Y}$  is contained within  $\mathbf{X}$ ; and  
10          if the first termination condition is TRUE, terminating and recording  
11  $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$  as a final bound.

1           14. (Original) The computer-readable storage medium of claim 13,  
2 wherein the method further comprises determining if  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular by  
3 computing a pre-conditioned Jacobian,  $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$ , wherein  $\mathbf{B}$  is an  
4 approximate inverse of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ , and then solving for the interval  
5 vector  $\mathbf{Y}$  that contains the value of  $\mathbf{y}$  that satisfies  $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$ , where  $\mathbf{r}(\mathbf{x})$   
6  $= -\mathbf{B}\mathbf{f}(\mathbf{x})$ .

1           15. (Original) The computer-readable storage medium of claim 14,  
2 wherein the method further comprises applying term consistency to  $\mathbf{B}\mathbf{f}(\mathbf{x}) = 0$ .

1           16. (Original) The computer-readable storage medium of claim 11,  
2 wherein if no termination condition is satisfied, the method further comprises  
3 returning to perform an interval Newton step on the interval vector  $\mathbf{Y}$ .

1           17. (Original) The computer-readable storage medium of claim 16,  
2 wherein returning to perform the interval Newton step on the interval vector  $\mathbf{Y}$  can  
3 involve splitting the interval vector  $\mathbf{X} = \mathbf{Y} \cap \mathbf{X}$ .

1           18. (Original) The computer-readable storage medium of claim 12,  
2 wherein the method further comprises:  
3           evaluating a second termination condition;  
4           wherein the second termination condition is TRUE if a function of the  
5 width of the interval vector  $\mathbf{X}$  is less than a pre-specified value,  $\varepsilon_X$ , and the  
6 absolute value of the function,  $\mathbf{f}$ , over the interval vector  $\mathbf{X}$  is less than a pre-  
7 specified value,  $\varepsilon_F$ ; and  
8           if the second termination condition is TRUE, terminating and recording  $\mathbf{X}$   
9 as a final bound.

1           19. (Original) The computer-readable storage medium of claim 11,  
2 wherein for each term,  $g(x_j)$ , that can be analytically inverted within the equation  
3  $f_i(\mathbf{x}) = 0$ , the method further comprises:  
4           setting  $X_j = X_j^+$  in  $\mathbf{X}$ ; and  
5           repeating the process of symbolically manipulating, substituting, solving  
6 and intersecting to produce the new interval vector  $X_j^+$ .

1           20. (Original) The computer-readable storage medium of claim 11,  
2 wherein symbolically manipulating  $f_i(\mathbf{x}) = 0$  involves selecting the invertible term  
3  $g(x_j)$  as the dominating term of the function  $f_i(\mathbf{x}) = 0$  within the interval vector  $\mathbf{X}$ .

1           21. (Currently amended) An apparatus that uses a computer system to  
2 solve a system of nonlinear equations specified by a vector function,  $\mathbf{f}$ , wherein

3  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  represents  $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \dots, f_n(\mathbf{x}) = 0$ , wherein  $\mathbf{x}$  is a  
 4 vector  $(x_1, x_2, x_3, \dots, x_n)$ , the apparatus comprising:  
 5 a receiving mechanism that is configured to receive a representation of an  
 6 interval vector  $\mathbf{X} = (X_1, X_2, \dots, X_n)$ , wherein for each dimension,  $i$ , the  
 7 representation of  $X_i$  includes a first floating-point number,  $a_i$ , representing the left  
 8 endpoint of  $X_i$ , and a second floating-point number,  $b_i$ , representing the right  
 9 endpoint of  $X_i$ ;  
 10 a symbolic manipulation mechanism, wherein for each nonlinear equation  
 11  $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$  in the system of equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ , the symbolic  
 12 manipulation mechanism is configured to manipulate  $f_i(\mathbf{x}) = 0$  to solve for any  
 13 invertible term,  $g(x'_j)$ , thereby producing a modified equation  $g(x'_j) = h(\mathbf{x})$ ,  
 14 wherein  $g(x'_j)$  ~~can be~~ is analytically inverted to produce an inverse function  $g^{-1}(\mathbf{y})$ ;  
 15 a solving mechanism that is configured to,  
 16 substitute the interval vector  $\mathbf{X}$  into the modified equation  
 17 to produce the equation  $g(X'_j) = h(\mathbf{X})$ , and to  
 18 solve for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and  
 19 an intersecting mechanism that is configured to intersect  $X'_j$  with the  
 20 vector element  $X_j$  to produce a new interval vector  $\mathbf{X}^+$ , wherein the new interval  
 21 vector  $\mathbf{X}^+$  contains all solutions of the system of equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  within the  
 22 interval vector  $\mathbf{X}$ , and wherein the width of the new interval vector  $\mathbf{X}^+$  is less than  
 23 or equal to the width of the interval vector  $\mathbf{X}$ .

1 22. (Original) The apparatus of claim 21, further comprising an interval  
 2 Newton mechanism that is configured to perform an interval Newton step on  $\mathbf{X}$  to  
 3 produce a resulting interval vector,  $\mathbf{Y}$ , wherein the point of expansion of the  
 4 interval Newton step is a point,  $\mathbf{x}$ , within the interval vector  $\mathbf{X}$ , and wherein  
 5 performing the interval Newton step involves evaluating  $\mathbf{f}(\mathbf{x})$  using interval  
 6 arithmetic to produce an interval result  $\mathbf{f}^l(\mathbf{x})$ .

1           23. (Original) The apparatus of claim 22, further comprising a termination  
2 mechanism that is configured to:  
3           evaluate a first termination condition, wherein the first termination  
4 condition is TRUE if,  
5                           zero is contained within  $\mathbf{f}^l(\mathbf{x})$ ,  
6                            $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular, wherein  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is the Jacobian of the  
7 function  $\mathbf{f}$  evaluated as a function of  $\mathbf{x}$  over the interval vector  $\mathbf{X}$ ,  
8                           and  
9                            $\mathbf{Y}$  is contained within  $\mathbf{X}$ ; and to  
10           wherein if the first termination condition is TRUE, the termination  
11 mechanism is configured to terminate and recording  $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$  as a final bound.

1           24. (Original) The apparatus of claim 23, wherein the termination  
2 mechanism is configured to determine if  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is regular by computing a pre-  
3 conditioned Jacobian,  $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$ , wherein  $\mathbf{B}$  is an approximate inverse of  
4 the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ , and then to solve for the interval vector  $\mathbf{Y}$  that contains the  
5 value of  $\mathbf{y}$  that satisfies  $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$ , where  $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$ .

1           25. (Original) The apparatus of claim 24, wherein the symbolic  
2 manipulation mechanism is additionally configured to apply term consistency to  
3  $\mathbf{B}\mathbf{f}(\mathbf{x}) = 0$ .

1           26. (Original) The apparatus of claim 21, wherein if no termination  
2 condition is satisfied, the apparatus is configured to return to perform an interval  
3 Newton step on the interval vector  $\mathbf{Y}$ .



1           27. (Original) The apparatus of claim 26, wherein returning to perform the  
2 interval Newton step on the interval vector  $\mathbf{Y}$  can involve splitting the interval  
3 vector  $\mathbf{X}=\mathbf{Y} \cap \mathbf{X}$ .

1           28. (Original) The apparatus of claim 22, wherein the termination  
2 mechanism that is configured to:  
3           evaluate a second termination condition;  
4           wherein the second termination condition is TRUE if a function of the  
5 width of the interval vector  $\mathbf{X}$  is less than a pre-specified value,  $\epsilon_X$ , and the  
6 absolute value of the function,  $\mathbf{f}$ , over the interval vector  $\mathbf{X}$  is less than a pre-  
7 specified value,  $\epsilon_F$ ; and  
8           wherein if the second termination condition is TRUE, the termination  
9 mechanism is configured to terminate and record  $\mathbf{X}$  as a final bound.

1           29. (Original) The apparatus of claim 21, wherein for each term,  $g(x_j)$ , that  
2 can be analytically inverted within the equation  $f_i(\mathbf{x}) = 0$ , the apparatus is  
3 configured to:  
4           set  $X_j = X_j^+$  in  $\mathbf{X}$ ; and to  
5           repeat the process of symbolically manipulating, substituting, solving and  
6 intersecting to produce the new interval vector  $X_j^+$ .

1           30. (Original) The apparatus of claim 21, wherein symbolically  
2 manipulating  $f_i(\mathbf{x})=0$  involves selecting the invertible term  $g(x_j)$  as the dominating  
3 term of the function  $f_i(\mathbf{x}) = 0$  within the interval vector  $\mathbf{X}$ .